

The only way to learn mathematics is to do mathematics.

— Paul Halmos —



## PROBABILITY

I hear and I forget.  
I see and I remember.  
I do and I understand.  
— Confucius —



## BASICS



- Experiment
- Sample Space
- Event
  - Simple or Elementary Event
  - Composite or Compound Event
- Event Space
- Probability of an Event
- Mutually Exclusive Events
- Mutually Exhaustive Events
- Equally Likely Events

## BASICS



### Note:

- The probability of an event lies between 0 and 1, both inclusive.  
i.e.  $0 \leq P(A) \leq 1$ .
- Non-occurrence of event A is denoted by  $A'$  or  $A^c$  or  $\bar{A}$  and it is known as **complimentary event of A**.
- The event A along with its complimentary  $A'$  forms a set of mutually exclusive and exhaustive events.  
i.e.  $P(A) + P(A') = 1 \Rightarrow P(A) = 1 - P(A')$

## EXAMPLES



**Example 1:** Determine the probability p, or an estimate of it, for each of the following events:

- i. An odd number appears in a single toss of a fair die.
- ii. At least one head appears in two tosses of a fair coin.
- iii. At least one head appears in three tosses of a fair coin.
- iv. An ace, 10 of diamonds, or 2 of spades appears in drawing a single card from a well-shuffled ordinary deck of 52 cards.
- v. The sum 7 appears in a single toss of a pair of fair dice.
- vi. A tail appears in the next toss of a coin if out of 100 tosses 56 were heads.

## EXAMPLES



**Example 2:** A coin is tossed three times. What is probability of getting

- a) 2 heads
- b) At least 2 heads

## EXAMPLES



**Example 3:** A dice is rolled twice. What is a probability of getting a difference of 2 points?

## EXAMPLES



**Example 4:** Two dice are thrown simultaneously. Find the probability that the sum of points on the two dice would be 7 or more.

## EXAMPLES



**Example 5:** Find the probability that a four digit number comprising the digits 2, 5, 6 and 7 would be divisible by 4.

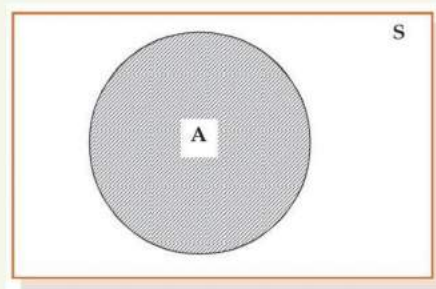
## EXAMPLES



**Example 6:** A committee of 4 members is to be formed from a group comprising 6 gentlemen and 4 ladies. What is the probability that the committee would comprise:

- i. 2 ladies
- ii. at least 1 ladies.

## SET-THEORETIC APPROACH

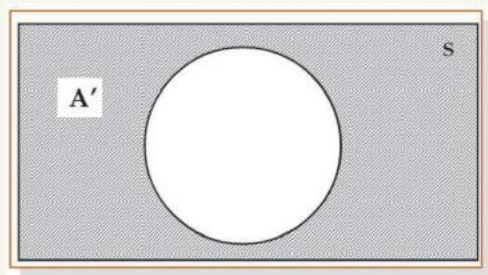


**Example:** A fair die is rolled. Let A be an event that even number appears.

Then  $S = \{1,2,3,4,5,6\}$  and  $A = \{2,4,6\}$ .



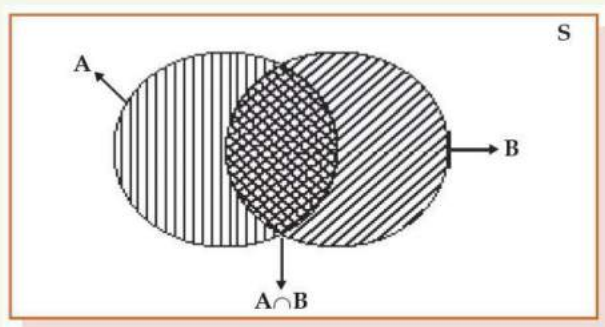
## SET-THEORETIC APPROACH



**Example:** A fair die is rolled. Let  $A$  be an event that even number appears.

Then  $S = \{1,2,3,4,5,6\}$ ,  $A = \{2,4,6\}$  and  $A' = \{1,3,5\}$ .

## SET-THEORETIC APPROACH



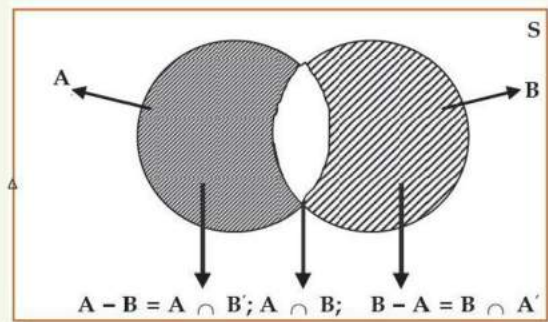
**Example:** A fair die is rolled. Let  $A$  be an event that even number appears and  $B$  be an event that prime number appears.

Then  $S = \{1,2,3,4,5,6\}$ ,  $A = \{2,4,6\}$  and  $B = \{2,3,5\}$

$A$  or  $B = A + B = A \cup B = \{2,3,4,5,6\}$

$A$  and  $B = AB = A \cap B = \{2\}$

## SET-THEORETIC APPROACH



**Example:** A fair die is rolled. Let A be an event that even number appears and B be an event that prime number appears.

Then  $S = \{1,2,3,4,5,6\}$ ,  $A = \{2,4,6\}$  and  $B = \{2,3,5\}$

*A but not B*  $= A - B = A/B = \{4,5\}$

*B but not A*  $= B - A = B/A = \{3,5\}$

## ADDITION THEOREMS



**Theorem 1:** For any two **mutually exclusive events** A and B, the probability that either A or B occurs is given by the sum of individual probabilities of A and B

i.e.  $P(A \text{ or } B) = P(A + B) = P(A \cup B) = P(A) + P(B)$ .

**Example 1:** A single card is drawn from an ordinary deck of cards. What is the probability that the selected card is an ace or a king?

**Example 2:** A pair of dice rolled. What is the probability that the sum of outcomes is 7 or 11?

**Example 3:** A number is selected from the first 25 natural numbers. What is the probability that it would be divisible by 4 or 7?

## ADDITION THEOREMS



**Theorem 2:** For any two events A and B, the probability that either A or B occurs is given by the sum of individual probabilities of A and B less the probability of simultaneous occurrence of the events A and B

$$\text{i.e. } P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

**Example 1:** A single card is drawn from an ordinary deck of cards. What is the probability that the selected card is an ace or a spade?

**Example 2:** A fair die is rolled. What is the probability that the outcome is odd or prime?

**Example 3:** A number is selected at random from the first 1000 natural numbers. What is the probability that it would be a multiple of 5 or 9?

## ADDITION THEOREMS



**Theorem 1:** For any two **mutually exclusive events** A and B, the probability that either A or B occurs is given by the sum of individual probabilities of A and B

$$\text{i.e. } P(A \text{ or } B) = P(A + B) = P(A \cup B) = P(A) + P(B).$$

**Theorem 2:** For any two events A and B, the probability that either A or B occurs is given by the sum of individual probabilities of A and B less the probability of simultaneous occurrence of the events A and B

$$\text{i.e. } P(A \cup B) = P(A) + P(B) - P(A \cap B)$$



## ADDITION THEOREMS



**Example 4:** The probability that an Accountant's job applicant has a B. Com. Degree is 0.85, that he is a CA is 0.30 and that he is both B. Com. and CA is 0.25 out of 500 applicants, how many would be B. Com. or CA?

**Example 5:** If  $P(A - B) = \frac{1}{5}$ ,  $P(A) = \frac{1}{3}$  and  $P(B) = \frac{1}{2}$ , what is the probability that out of the two events A and B, only B would occur?

**Example 6:** In a survey of a group of people it was found that 25% were smokers and drinkers, 10% were smokers but not drinkers, and 35% were drinkers but not smokers. What percent in the survey were either smokers or drinkers or both?

## ADDITION THEOREMS



**Theorem 3:** For any three events A, B and C, the probability that at least one of the events occurs is given by

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

**Example :** A large software development company employs 100 computer programmers. Of them, 45 are proficient in Java, 30 in C#, 20 in Python, six in C# and Java, one in Java and Python, five in C# and Python, and just one programmer is proficient in all three languages above. Find the probability that the randomly selected employee

- is proficient in at least one language.
- is not proficient in any of these three languages.

## CONDITIONAL PROBABILITY



If A and B are two events, and the occurrence of one event, say B, is influenced by the occurrence of another event A, then the two events A and B are known as **dependent events**.

The probability that B occurs given that A has occurred is denoted by  $P(B|A)$ , and is called the conditional probability of **B given that A** has occurred. This is given by

$$\begin{aligned} P(A \cap B) &= P(A)P(B|A) \\ \Rightarrow P(B|A) &= \frac{P(A \cap B)}{P(A)}, \quad \text{provided } P(A) \neq 0. \end{aligned}$$

**Example 1:** Two cards are drawn from a well-shuffled ordinary deck of 52 cards without replacement. Find the probability that they are both aces.

**Example 2:** A purse contains 2 silver coins and 4 copper coins, and a second purse contains 4 silver coins and 3 copper coins. If a coin is selected at random from one of the two purses, what is the probability that it is a silver coin?

## INDEPENDENT EVENTS



If the occurrence of the second event B is not influenced by the occurrence of the first event A, then B is known to be **independent** of A.

It also follows that in this case, A is also independent of B and A and B are known as **mutually independent**.

In this case, we get  $P(B|A) = P(B)$  and also  $P(A|B) = P(A)$   
 $\Rightarrow P(A \cap B) = P(A)P(B)$

Also note that if the two events **A and B are independent** then the following pairs of events are also independent:

- i. **A and B'**
- ii. **A' and B**
- iii. **A' and B'**

## INDEPENDENT EVENTS



**Example 1:** Two cards are drawn from a well-shuffled ordinary deck of 52 cards with replacement. Find the probability that they are both aces.

**Example 2:** The odds in favour of A winning a game of chess against B are 3 : 2. If three games are to be played, what are the odds

- in favour of A's winning at least two games out of the three and
- against A losing the first two games to B?

## THEOREMS OF COMPOUND PROBABILITY



**Theorem 4:** For an two events A and B, the probability that A and B occur simultaneously is given by  $P(A \cap B) = P(A)P(B|A)$ , provided  $P(A) > 0$ .

**Theorem 5:** For any three event A, B and C, the probability that they occur jointly is given by  $P(A \cap B \cap C) = P(A)P(B|A)P(C|A \cap B)$ , provided  $P(A \cap B) > 0$ .

In case events A, B and C are mutually independent,

$$P(A \cap B \cap C) = P(A)P(B)P(C)$$



## THEOREMS OF COMPOUND PROBABILITY



**Example 1:** Three balls are drawn successively from a box containing 6 red balls, 4 white balls, and 5 blue balls. Find the probability that they are drawn in the order red, white, and blue if each ball is

- i. replaced and
- ii. not replaced

## THEOREMS OF COMPOUND PROBABILITY



**Example 2:** The probability of A hitting a target is  $\frac{2}{3}$  and that of B is  $\frac{4}{5}$ . they both fire at the target. What is the probability that target would be hit? What is the probability that only one of them will hit the target?



## THEOREMS OF COMPOUND PROBABILITY



**Example 4:** In connection with a random experiment, it is found that

$$P(A) = \frac{2}{3}, P(B) = \frac{3}{5} \text{ and } P(A \cup B) = \frac{5}{6}.$$

Evaluate (i)  $P(A|B)$  (ii)  $P(B|A)$  (iii)  $P(A'|B)$  (iv)  $P(A|B')$  (v)  $P(A'|B')$ .

## BAYES' THEOREM



**THEOREM:** Let  $E_1, E_2, E_3, \dots, E_n$  be mutually exclusive and exhaustive events. Let  $A$  be any event. Then

$$P[E_k/A] = \frac{P[E_k]P[A/E_k]}{\sum_{i=1}^n P[A/E_i]}$$

## BAYES' THEOREM



**Example 1:** Three identical urns contains red and blue marbles. The first urn contains 4 red and 5 blue marbles, the second urn contains 2 red and 4 blue marbles and the third urn contains 5 red and 3 blue marbles. An urn is chosen randomly and a marble is drawn from it. If the marble drawn is red, what will be the probability that the second urn is chosen?

## BAYES' THEOREM



**Example 2:** In a city, 54% of the adults are males. Also 20% of males are smokers and 12% of females are smokers. One adult is randomly selected from the population. If the selected person is a smoker then what is the probability that he is male?

## BAYES' THEOREM



**Example 3:** In a survey, it was found that out of 100 people 4 has disease. If test has 2% false negative rate and 5% false positive rate. Find

- i) the probability that a person who has disease, tests positive
- ii) the probability that a person who has disease, tests negative
- iii) the probability that a person who is healthy, tests positive
- iv) the probability that a person who is healthy, tests negative
- v) the probability that a person tests positive actually has disease
- vi) the probability that a person tests negative is actually healthy



Thank you