



The only way to learn mathematics is to do mathematics.

— Paul Halmos —



COUNTING PRINCIPLE

I hear and I forget.
I see and I remember.
I do and I understand.

— Confucius —



BASIC



Sum Rule Principle: Suppose some event E can occur in m ways and a second event F can occur in n ways, and suppose both events cannot occur simultaneously. Then E or F can occur in $m + n$ ways.

Product Rule Principle: Suppose there is an event E which can occur in m ways and, independent of this event, there is a second event F which can occur in n ways. Then combinations of E and F can occur in mn ways.

BASIC



- 1) Suppose a bookcase shelf has 6 mathematics textbooks, 5 programming textbooks and 3 networking textbooks. Find the number of ways a student can choose a textbook.

BASIC



- 2) Suppose a bookcase shelf has 6 mathematics textbooks, 5 programming textbooks and 3 networking textbooks. Find the number of ways a student can choose one of each type of textbook.

BASIC



- 3) A class has 10 male students and 8 female students. Find the number of ways the class can elect: (a) a class representative; (b) two class representatives, one male and one female.

BASIC



- 4) A lock code consists of two letters followed by two digits. How many distinct lock codes are possible if
- i) repetition allowed
 - ii) repetition not allowed

BASIC



Factorial Notation:

For all $n \in \mathbb{Z}^+$

$$n! = n \times (n-1) \times (n-2) \times \dots \times 3 \times 2 \times 1$$

We define $0! = 1$.

$$\therefore (n-1)! = (n-1) \times (n-2) \times \dots \times 3 \times 2 \times 1$$

Example:

$$\therefore n! = n \times (n-1)!$$

$$1! = 1$$

We can also write,

$$\therefore n! = n \times (n-1) \times (n-2)!$$

$$2! = 2 \times 1 = 2$$

$$3! = 3 \times 2 \times 1 = 6$$

$$4! = 4 \times 3 \times 2 \times 1 = 24$$

$$5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$$

PERMUTATIONS



Any arrangement of r objects out of n objects, ($r \leq n$), is called an “ r -permutation” or “*permutation of the n objects taken r at a time*”.

It is denoted by nPr or $P(n, r)$.

$$P(n, r) = \frac{n!}{(n-r)!}.$$

Note:

$$1) \quad nP0 = 1$$

$$2) \quad nP1 = n$$

$$3) \quad nPn = n!$$

PERMUTATIONS



Example:

$${}_5P_0 = 1$$

$$\text{using } {}_nP_0 = 1$$

$${}_7P_1 = 7$$

$$\text{using } {}_nP_1 = n$$

$${}_6P_6 = 6! = 720$$

$$\text{using } {}_nP_n = n!$$

$${}_7P_2 = \frac{7!}{(7-2)!}$$

$$\text{using } {}_nP_r = \frac{n!}{(n-r)!}$$

$$= \frac{7!}{5!} = \frac{7 \times 6 \times 5!}{5!} = 7 \times 6 = 42$$

$${}_{10}P_3 = 10 \times 9 \times 8 = 720$$

$${}_7P_4 = 7 \times 6 \times 5 \times 4 = 840$$

PERMUTATIONS



- 1) How many three digit numbers can be formed out of the digits 0, 1, 3, 5, 7, 9 if no digit is repeated in any number? How many of these will be greater than 500?

PERMUTATIONS



- 2) How many different words can be formed out of the letters of the word 'COMPUTER'?

PERMUTATIONS



- 3) How many different words beginning and ending with a vowel can be formed out of the letters of the word 'HARMONIC'?

PERMUTATIONS



- 4) Four boys and five girls are made to stand in a line.
- a) How many different arrangements can be made so that, no two boys are together?
 - b) How many different arrangements can be made so that, all four boys are always together?

PERMUTATIONS



- 5) In how many ways can 5 mathematics books, 4 programming books and 2 networking books be arranged on a shelf, if the books on the same subject are to be together?

PERMUTATION WITH REPETITIONS



The number of permutations of n objects of which n_1 are alike, n_2 are alike,, n_r are alike, is given by

$$P(n; n_1, n_2, \dots, n_r) = \frac{n!}{n_1! n_2! \dots n_r!}$$

PERMUTATION WITH REPETITIONS



1) Find the number of seven letter words that can be formed using the letters of the word 'BENZENE'.

PERMUTATION WITH REPETITIONS



2) Find the number of seven letter words that can be formed using the letters of the word 'UNUSUAL'.

PERMUTATION WITH REPETITIONS



3) Find the number of twelve letter words that can be formed using the letters of the word 'SOCIOLOGICAL'.

COMBINATIONS



A **combination** of n elements taken r at a time is any **selection** of r elements where order does not count. Such a selection is called an **r -combination**.

It is denoted by nCr or $C(n, r)$.

$$nCr = C(n, r) = \frac{n!}{(n-r)! r!}.$$

Note:

- 1) $nC0 = 1$
- 2) $nC1 = n$
- 3) $nCn = 1$
- 4) $nCr = nC(n - r)$

COMBINATIONS



Example:

$$5C0 = 1$$

$$\text{using } nC0 = 1$$

$$7C1 = 7$$

$$\text{using } nC1 = n$$

$$6C6 = 1$$

$$\text{using } nCn = 1$$

$$7C2 = \frac{7!}{(7-2)! 2!}$$

$$\text{using } nCr = \frac{n!}{(n-r)! r!}$$

$$= \frac{7!}{5! 2!} = \frac{7 \times 6 \times 5!}{5! 2!} = \frac{7 \times 6}{2} = 21$$

$$10C3 = \frac{10 \times 9 \times 8}{3!} = 120$$

$$7C5 = 7C2 = \frac{7 \times 6}{2!} = 21 \quad \text{using } nCr = nC(n - r)$$

COMBINATIONS



- 1) A class contains 10 students with 6 boys and 4 girls. Find the number of ways to:
- a) Select a 4-member committee
 - b) Select a 4-member committee with 2 boys and 2 girls

COMBINATIONS



- 2) A box contains 8 blue socks and 6 red socks. Find the number of ways two socks can be drawn from the box if:
- a) They can be any colour
 - b) They must be the same colour.

COMBINATIONS



- 3) A class contains 8 boys and 4 girls. Find the number of ways a teacher can select a committee of 4 from the class where there is:
- a) no restrictions
 - b) 2 boys and 2 girls
 - c) At least 1 girl

COMBINATIONS



- 4) A committee contains 8 men and 4 women members. Find the number of ways a president, a vice-president and a treasurer can be selected for the committee.

MISCELLANEOUS



- a.
 - i. How many five-digit integers (integers from 10,000 through 99,999) are divisible by 5?
 - ii. What is the probability that a five-digit integer chosen at random is divisible by 5?
- b.
 - i. If any seven digits could be used to form a telephone number, how many seven-digit telephone numbers would not have any repeated digits?
 - ii. How many seven-digit telephone numbers would have at least one repeated digit?
 - iii. What is the probability that a randomly chosen seven-digit telephone number would have at least one repeated digit?
- c.
 - i. How many distinguishable ways can the letters of the word MILLIMICRON be arranged in order?
 - ii. How many distinguishable orderings of the letters of MILLIMICRON begin with M and end with N?
 - iii. How many distinguishable orderings of the letters of MILLIMICRON contain the letters CR next to each other in order and also the letters ON next to each other in order?

INCLUSION EXCLUSION PRINCIPLE



For two sets:

Let A and B be any two finite sets, then

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

For three sets:

Let A, B and C be any three finite sets, then

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$$

INCLUSION EXCLUSION PRINCIPLE



Q.1 In a class of 50 students, 25 like Maths and 15 like Programming, 10 like Maths and Programming. Find

- a) How many like at least one of them?
- b) How many do not any of the subjects?

INCLUSION EXCLUSION PRINCIPLE



Q.2 Out of 150 residents of a building, 105 speak Marathi, 75 speak Gujarati and 45 speak both the languages. Find the number of residents who do not speak either of the languages also find the number of residents who speak only Marathi.

INCLUSION EXCLUSION PRINCIPLE



Q.3 In a survey of people it was found that 80 people watch football, 60 watch cricket, 50 watch hockey, 30 watch football and cricket, 20 watch football and hockey, 15 watch cricket and hockey and 10 watch all three games.

- a) How many people watch at least one game?
- b) How many people watch only cricket?
- c) How many people watch football and cricket but not hockey?

INCLUSION EXCLUSION PRINCIPLE



Q.4 In a survey of 120 people, it was found that:

65 read Newsweek magazine, 45 read Time, 42 read Fortune,
20 read both Newsweek and Time, 25 read both Newsweek and Fortune,
15 read both Time and Fortune, 8 read all the three magazines.

- (a) Find the number of people who read at least one of the three magazines.
- (b) Find the number of people who read exactly one magazine.

